EXERCISE 2.4

Q1. Use laws of exponents to simplify:

(i) $\dfrac{(243)^{-2/3}\,(32)^{-1/5}}{\sqrt{(196)^{-1}}}$

Solution:

$$= \frac{(243)^{-2/3}\,(32)^{-1/5}}{(196)^{-\frac{1}{2}}}$$

$$= \left(\frac{1}{243}\right)^{2/3} \times \left(\frac{1}{32}\right)^{1/5} \times (196)^{1/2}$$

$$= \left(\frac{1}{3^5}\right)^{2/3} \times \left(\frac{1}{2^5}\right)^{1/5} \times (4 \times 49)^{1/2}$$

$$= \frac{1}{3^{10/3}} \times \frac{1}{2^{5/5}} \times (2^2)^{1/2} \times (7^2)^{1/2}$$

$$= \frac{1}{3^{1/3} \times 3^{9/3}} \times \frac{1}{2} \times 2 \times 7 \qquad \left(\because \frac{1}{3^{10/3}} = \frac{1}{3^{1/3} \times 3^{9/3}}\right)$$

$$= \frac{1}{3^3 \times 3^{1/3}} \times 7$$

$$= \frac{7}{3^3 \times \sqrt[3]{3}} = \frac{7}{27\left(\sqrt[3]{3}\right)}$$

(ii) $\left(2x^5y^{-4}\right)\left(-8x^{-3}\,y^2\right)$

Solution:

$$= 2 \times (-8) \times x^5 \cdot x^{-3} \cdot y^{-4} \cdot y^2$$

$$= -16x^{5-3} \cdot y^{-4+2} = -16x^2\,y^{-2}$$

$$= -\frac{16x^2}{y^2}$$

(iii) $\left(\dfrac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0}\right)^{-3}$

Solution:

$$= \left(\frac{x^4y^{-3}z^0}{x^{-2}y^{-1}z^{-4}}\right)^3 = \left(\frac{x^{4+2} \cdot z^{0+4}}{y^{3-1}}\right)^3$$

$$= \left(\frac{x^6 \cdot z^4}{y^2}\right)^3 = \frac{x^{6\times3} \cdot z^{4\times3}}{y^{2\times3}}$$

$$= \frac{x^{18}\, z^{12}}{y^6}$$

(iv) $\dfrac{(81)^n \cdot 3^5 - (3)^{4n-1}(243)}{(9^{2n})(3^3)}$

Solution:

$$= \frac{(3^4)^n \cdot 3^5 - (3)^{4n-1} \cdot 3^5}{(3^2)^{2n}\,(3^3)} = \frac{3^{4n} \cdot 3^5 - 3^{4n-1} \cdot 3^5}{3^{4n} \cdot 3^5}$$

$$= \frac{3^{4n+5} - 3^{4n-1+5}}{3^{4n+3}} = \frac{3^{4n+5} - 3^{4n+4}}{3^{4n+3}}$$

$$= \frac{3^{4n+4}(3-1)}{3^{4n+3}} = 3^{4n+4-4n-3} \cdot (2)$$

$$= (3) \times (2) = 6$$

Q2. Show that $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} = 1$

Solution:

$$= \left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$$

$$= (x^a \cdot x^{-b})^{a+b} \times (x^b \cdot x^{-c})^{b+c} \times (x^c \cdot x^{-a})^{c+a}$$

$$= \left(x^{a-b}\right)^{a+b} \times \left(x^{b-c}\right)^{b+c} \times (x^{c-a})^{c+a}$$

$$= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$$

$$= x^{a^2-b^2+b^2-c^2+c^2-a^2}$$

$$= x^0$$

$$= 1$$

Q3. Simplify

(i) $\dfrac{2^{1/3} \times (27)^{1/3} \times (60)^{1/2}}{(180)^{1/2} \times (4)^{-1/3} \times (9)^{1/4}}$

Solution:

$$= \frac{2^{1/3} \times (3^3)^{1/3} \times (3 \cdot 5 \cdot 2^2)^{1/2}}{(2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5)^{1/2} \times (2^2)^{-1/3} \times (3^2)^{1/4}}$$

$$= \frac{2^{1/3} \cdot 3^{3\times 1/3} \cdot 3^{1/2} \cdot 5^{1/2} \cdot 2^{2\times 1/2}}{(2^2 \cdot 3^2 \cdot 5)^{1/2} \cdot 2^{-2/3} \cdot 3^{2/4}}$$

$$= \frac{2^{1/3} \cdot 3 \cdot 3^{1/2} \cdot 5^{1/2} \cdot 2}{(2^2 \cdot 3^2 \cdot 5)^{1/2} \cdot 2^{-2/3} \cdot 3^{2/4}}$$

$$= \frac{2^{1/3} \cdot 3^{1/2} \cdot 5^{1/2} \cdot 3 \cdot 2}{2 \cdot 3 \cdot 5^{1/2} \cdot 2^{-2/3} \cdot 3^{1/2}}$$

$$= \frac{2^{\frac{1}{3}+\frac{2}{3}} \cdot 3^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} \cdot 3 \cdot 2}{5^{\frac{1}{2}}\ 3^{\frac{1}{2}}\ 2\ 3}$$

$$= 2^{3/3}$$

$$= 2^1 = 2$$

(ii) $\sqrt{\frac{(216)^{2/3}\times(25)^{1/2}}{(0.04)^{-1/2}}}$

Solution:

$$= \sqrt{\frac{(2^3 \cdot 3^3)^{2/3} \cdot (5^2)^{1/2}}{\left(\frac{4}{100}\right)^{-1/2}}}$$

$$= \sqrt{2^2 \quad 3^2 \quad 5 \quad \left(\frac{4}{100}\right)^{1/2}}$$

$$= \sqrt{2^2 \cdot 3^2 \cdot 5 \cdot \left(\frac{2^2}{10^2}\right)^{1/2}}$$

$$= \sqrt{2^2 \cdot 3^2 \cdot 5 \cdot \left(\frac{2}{10}\right)^{2\times\frac{1}{2}}}$$

$$= \sqrt{2^2 \quad 3^2 \cdot 5 \cdot \frac{2}{10}}$$

$$= \sqrt{2^2 \cdot 3^2}$$

$$= 2 \quad 3$$

$$= 6$$

(iii) $5^2 \div (5^2)^3$

Solution:

$$= 5^8 \div 5^6 \quad = \quad \frac{5^8}{5^6} \quad = \quad 5^{8-6}$$

$$= 5^2 \quad = \quad 25$$

(iv) $(x^3)^2 \div x^{3^2} \,.\quad x \neq 0$

Solution:

$$= x^{3\times2} \div x^{3\times3} \quad = \quad x^6 \div x^9 \quad = \quad \frac{x^6}{x^9}$$

$$= x^{6-9} \quad = \quad x^{-3} \quad = \quad \frac{1}{x^3}$$